Optimal Intercept Guidance for Short-Range Tactical Missiles

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Introduction

Missile dynamic time lags, guidance command saturation, and target acceleration are major factors contributing to excessive terminal miss distances resulting from short-range intercept trajectories. The objective of this Technical Note is to describe a linear closed loop guidance law which compensates for these factors. Optimal control theory is utilized where the missile dynamics, represented as a single time lag, and target acceleration are defined in the constraint equations. A quadratic performance index is employed which implicitly effects a "soft" limit on the acceleration command. A zero terminal miss distance is the only boundary constraint imposed on the problem. The final form of the guidance law which includes a time varying navigation gain represents an extension of the guidance laws discussed by Bryson,[1] Garber,[2] and Abzug[3] and is applicable to short range air-to-ground and air-to-air interception.

Optimal Guidance Law Derivation

A concise treatment to a special class of optimal control problems is presented by Bryson and Ho.[4] The problem is to minimize the quadratic performance index,

$$J = \frac{1}{2}\int_{t_0}^{t_f} u^2 dt \tag{1}$$

subject to the linear homogeneous differential equations of constraint,

$$\dot{\mathbf{X}} = \mathbf{F}(t)\mathbf{X} + \mathbf{G}(t)u \tag{2}$$

and the zero terminal miss distance boundary constraint

$$x(t_f) = 0 \tag{3}$$

It is assumed that the initial state, $\mathbf{X}(t_0)$, is given. The solution for the optimal closed loop control (guidance) law is

$$u = (\mathbf{G}^T\mathbf{P}/q)M \tag{4}$$

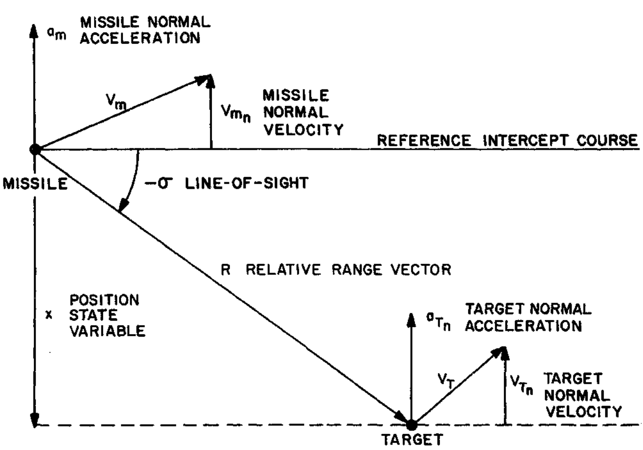

Fig. 1 Intercept geometry.

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where M is the predicted terminal miss distance when $u = 0$ and is given by

$$M = \mathbf{P}^T\mathbf{X} \tag{5}$$

Relative motion between the target and missile is considered with the intercept geometry shown in Fig. 1. The equations of motion are expressed in terms of state variables normal to the reference intercept course. The differential equations of constraint are

$$\dot{x} = v,\ \dot{v} = a_m - a_{T_n},\ \dot{a}_m = -(1/\tau)a_m + (1/\tau)u \tag{6}$$

where $v = V_{m_n} - V_{T_n}$. The missile dynamics are represented with a single time lag τ.

It is first assumed that the target acceleration is zero ($a_{T_n} = 0$) in order to comply with the homogeneity requirement of Eq. (2). Thus,

$$\mathbf{X} = \begin{bmatrix} x \\ v \\ a_m \end{bmatrix};\quad \mathbf{F} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau \end{bmatrix};\quad \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 1/\tau \end{bmatrix}$$

The solution for $\mathbf{P}$ is obtained from

$$\dot{\mathbf{P}} + \mathbf{F}^T\mathbf{P} = 0;\quad \mathbf{P}(t_f) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

and is found to be

$$\mathbf{P} = \begin{bmatrix} 1 \\ t_g \\ \tau^2(e^{-T} + T - 1) \end{bmatrix} \tag{8}$$

where t_g is the time-to-go defined as

$$t_g = t_f - t \tag{9}$$

and $T = t_g/\tau$. The parameter q is given by

$$q = -\int_t^{t_f} (\mathbf{P}^T\mathbf{G}\mathbf{G}^T\mathbf{P})dt \tag{10}$$

which becomes

$$q = \tau^3(\tfrac{1}{2}e^{-2T} + 2Te^{-T} - T^3/3 + T^2 - T - \tfrac{1}{2}) \tag{11}$$

The optimal closed loop guidance law given by Eq. (4) can now be expressed as

$$u = (\Lambda/t_g^2)[x + t_g v + \tau^2(e^{-T} + T - 1)a_m] \tag{12}$$

with the navigation gain given by

$$\Lambda = \frac{e^{-T} + T - 1}{(1/2T^2)e^{-2T} + (2/T)e^{-T} - T/3 + 1 - 1/T - 1/2T^2} \tag{13}$$

Note that

$$\lim_{T\to\infty} \Lambda = -3 \text{ and } \lim_{T\to 0} \Lambda = 0$$

Equation (12) suggests that the general form of the optimal guidance law is

$$u = (\Lambda/t_g^2)M \tag{14}$$

where

$$M = x + t_g v + \tau^2(e^{-T} + T - 1)a_m \tag{15}$$

for a nonaccelerating target. The optimal guidance law for an accelerating target can be constructed through consideration of the intercept kinematics rather than by derivations involving the nonhomogeneous constraint equations of Eq. (6). The predicted miss distance as a function of target acceleration becomes

$$M = x + t_g v + \tau^2(e^{-T} + T - 1)a_m - (t_g^2/2)a_{T_n} \quad (16)$$

Air-to-Ground Interception

The optimal closed loop guidance law given by Eqs. (14) and (16) is rewritten as

$$u = (\Lambda/t_g^2)(x + t_g v) + (\Lambda/T^2) \times (e^{-T} + T - 1)a_m - (\Lambda/2)a_{T_n} \quad (17)$$

It is desirable to express the state variables x and v in terms of parameters most commonly employed in tactical missile guidance design. If it is assumed that the reference intercept course is at any instant aligned with the line-of-sight, then

$$x = 0, \quad v = -R\dot{\sigma} \quad (18)$$

With the time-to-go approximated as

$$t_g = -R/\dot{R} \quad (19)$$

the optimal guidance law can be expressed as

$$u = \Lambda\dot{R}\dot{\sigma} + (\Lambda/T^2)(e^{-T} + T - 1)a_m - (\Lambda/2)a_{T_n} \quad (20)$$

Equation (20) is a biased proportional navigation guidance law where a time varying navigation gain and an acceleration feedback path provide compensation for the missile time lag. The target acceleration is the component of gravity normal to the line-of-sight. Figure 2 displays a block diagram representation of the air-to-ground guidance law.

Air-to-Air Interception

The quantity $[(\Lambda/2)a_m]$ is added to and subtracted from Eq. (17) to obtain

$$u = (\Lambda/t_g^2)[x + t_g v + (t_g^2/2)a] + (\Lambda/T^2)(e^{-T} + T - 1)a_m - (\Lambda/2)a_m \quad (21)$$

where

$$a = a_m - a_{T_n} \quad (22)$$

For the case where the reference intercept course is defined as the instantaneous line-of-sight, Eq. (22) may be written as

$$a = -2\dot{R}\dot{\sigma} - R\ddot{\sigma} \quad (23)$$

and the acceleration command normal to the line-of-sight becomes

$$u = (\Lambda\dot{R}t_g/2)\ddot{\sigma} + (\Lambda/T^2)(e^{-T} + T - 1)a_m - (\Lambda/2)a_m \quad (24)$$

A block diagram of the air-to-air guidance law is shown in Fig. 3.

Comments on the Optimal Guidance Laws

One of the most successfully employed tactical missile guidance techniques is proportional navigation. Trajectory simulations of simplified missile-target dynamics have shown that the terminal miss distance is substantially reduced for missiles employing the optimal guidance laws compared to miss distances resulting from proportional navigation. The air-to-ground optimal law has been incorporated into a high-order nonlinear missile dynamics simulation which yielded equally substantial performance improvements in terms of reduced miss distances and the infrequent occurrence of acceleration command saturation. The practicality of considering higher order missile dynamics in the guidance law synthesis is questionable when hardware implementation is involved. Each additional state variable defined in the differential equations of constraint appears in the guidance law as a time varying feedback term. The implication is that significant miss distance reductions are attainable through the optimal guidance laws when high-order missile dynamics are represented by a single time lag. Time-to-go estimations also introduce implementation problems. For missiles equipped with infrared and electrooptical sensing devices, passive time-to-go estimations such as those described by Rawling[5] are most promising.

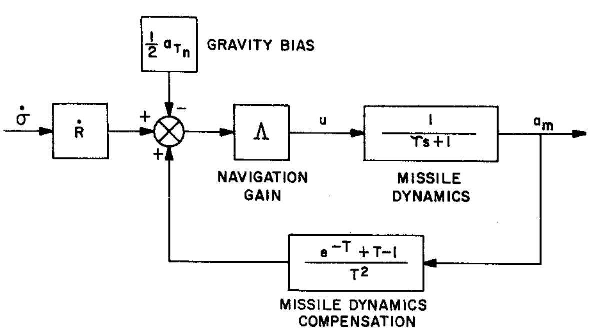

Fig. 2 **Short-range air-to-ground guidance law.**

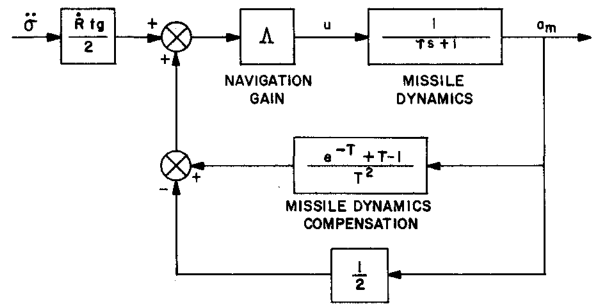

Fig. 3 **Short-range air-to-air guidance law.**

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Recovery Factor for Highly Accelerated Laminar Boundary Layers

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Nomenclature

C = $\rho\mu/\rho_e\mu_e$
E = $u_e^2/2H_e$, Mach number, or flow kinetic energy, parameter
f = dimensionless stream function
g = H/H_e

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